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23. Show that *f* (*x*) = |*x* − 5| is continuous but not differentiable at *x* = 5.

Solution:

Given, $f(x) = |x-5|$

$$\Rightarrow \quad f(x) = \begin{cases} -(x-5) \text{ if } x-5<0 \text{ or } x<5 \\ x-5 \text{ if } x-5>0 \text{ or } x>5 \end{cases}$$

For continuity at $x = 5$

$$\text{L.H.L. } \lim_{h \to 5^-} f(x) = -(x-5)$$

$$= \lim_{h \to 0} -(5-h-5) = \lim_{h \to 0} h = 0$$

$$\text{R.H.L. } \lim_{x \to 5^+} f(x) = x-5$$

$$= \lim_{h \to 0} (5+h-5) = \lim_{h \to 0} h = 0$$

$$\text{L.H.L.} = \text{R.H.L.}$$

So, $f(x)$ is continuous at $x = 5$.

Now, for differentiability

$$Lf'(5) = \lim_{h \to 0} \frac{f(5-h)-f(5)}{-h}$$

$$= \lim_{h \to 0} \frac{-(5-h-5)-(5-5)}{-h} = \lim_{h \to 0} \frac{h}{-h} = -1$$

$$Rf'(5) = \lim_{h \to 0} \frac{f(5+h)-f(5)}{h}$$

$$= \lim_{h \to 0} \frac{(5+h-5)-(5-5)}{h} = \lim_{h \to 0} \frac{h-0}{h} = 1$$

Thus, $\quad Lf'(5) \neq Rf'(5)$

Therefore, f(x) is not differentiable at x = 5.

24. A function *f* : R ® R satisfies the equation *f* (*x* + *y*) = *f* (*x*) *f* (*y*) for all *x*, *y* ÎR, *f* (*x*) ¹ 0. Suppose that the function is differentiable at *x* = 0 and *f* ¢ (0) = 2. Prove that *f* ¢(*x*) = 2 *f* (*x*).

Solution:

Given,

f : R ® R satisfies the equation *f* (*x* + *y*) = *f* (*x*) *f* (*y*) for all *x*, *y* ÎR, *f* (*x*) ¹ 0

Let us take any point x = 0 at which the function f(x) is differentiable.

$$\text{So, } f'(0) = \lim_{h \to 0} \frac{f(0+h) - f(0)}{h}$$

$$2 = \lim_{h \to 0} \frac{f(0).f(h) - f(0)}{h} \quad [\because f(0) = f(h)] \qquad ...(i)$$

$$\Rightarrow \quad 2 = \lim_{h \to 0} \frac{f(0)[f(h) - 1]}{h}$$

$$\text{Now, } f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{f(x).f(h) - f(x)}{h} \quad [\because f(x+y) = f(x).f(y)]$$

$$= \lim_{h \to 0} \frac{f(x)[f(h) - 1]}{h} = 2f(x) \qquad \text{from eqn. } (i)$$

$$\text{So, } f'(0) = \lim_{h \to 0} \frac{f(0+h) - f(0)}{h}$$

$$2 = \lim_{h \to 0} \frac{f(0).f(h) - f(0)}{h} \quad [\because f(0) = f(h)] \qquad \ldots(i)$$

$$\Rightarrow \quad 2 = \lim_{h \to 0} \frac{f(0)\left[f(h) - 1\right]}{h}$$

$$\text{Now, } f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{f(x).f(h) - f(x)}{h} \quad \left[\because f(x+y) = f(x).f(y)\right]$$

$$= \lim_{h \to 0} \frac{f(x)[f(h) - 1]}{h} = 2f(x) \qquad \text{from eqn. } (i)$$

Therefore, $f'(x) = 2f(x)$.

Differentiate each of the following w.r.t. *x* (Exercises 25 to 43) :

25. $2^{\cos^2 x}$

Solution:

Let $y = 2^{\cos^2 x}$

Taking log on both sides, we get

$$\log y = \log 2^{\cos^2 x} \Rightarrow \log y = \cos^2 x \,.\, \log 2$$

Now,

Differentiating both sides w.r.t. x

$$\frac{1}{y}.\frac{dy}{dx} = \log 2.\frac{d}{dx}\cos^2 x$$

$$\frac{1}{y}.\frac{dy}{dx} = \log 2\left[2\cos x.\frac{d}{dx}\cos x\right]$$

$$\frac{1}{y}.\frac{dy}{dx} = \log 2\ [2\cos x\,(-\sin x)]$$

$$\frac{1}{y}.\frac{dy}{dx} = \log 2\,(-\sin 2x)$$

$$\frac{dy}{dx} = -y\,.\,\log 2 \sin 2x$$

Thus, $$\frac{dy}{dx} = -2^{\cos^2 x}(\log 2 \sin 2x)$$

26.

$$\frac{8^x}{x^8}$$

Solution:

Let $y = \frac{8^x}{x^8}$

Taking log on both sides, we get $\log y = \log \frac{8^x}{x^8}$

$\Rightarrow \log y = \log 8^x - \log x^8 \Rightarrow \log y = x \log 8 - 8 \log x$

Differentiating both sides w.r.t. x

$\frac{1}{y} \cdot \frac{dy}{dx} = \log 8.1 - \frac{8}{x} \quad \Rightarrow \frac{dy}{dx} = y\left[\log 8 - \frac{8}{x}\right]$

Thus, $\frac{dy}{dx} = \frac{8^x}{x^8}\left[\log 8 - \frac{8}{x}\right]$